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EXPERIMENTAL STUDY OF TEMPERATURE DISTRIBUTION IN LAMINAR TUBE FLOW OF A FLUID WITH INTERNAL HEAT GENERATION

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Abstract—An experimental investigation of laminar fluid flow in a circular tube with internal heat generation was conducted. Wall-temperature measurements were made for a uniform internal heat source in an insulated tube, and excellent agreement was demonstrated between the experimental results and a theoretical analysis by Siegel *et al.*

NOMENCLATURE

A,	cross-sectional area of tube, πr_0^2;
C_u,	coefficient occurring in equation (2);
C_p,	specific heat at constant pressure;
D,	diameter, $2r_0$;
E,	voltage;
$f(r)$,	dimensionless velocity, $u(r)/\bar{u}$;
I,	current flow;
L,	length of test section;
Pr,	Prandtl modulus, $\mu C_p/\kappa$;
Q,	rate of internal heat generation per unit volume;
Re,	Reynolds number, $\rho \bar{u} D/\mu$;
R_u,	eigenfunctions of equation (2);
r,	radial co-ordinate measured from tube centerline;
r_0,	tube radius;
T,	temperature;
T_0,	inlet temperature;
T_b,	bulk-fluid temperature;
T_w,	tube-wall temperature;
u,	fluid velocity in x-direction;
$\bar{u}$,	mean fluid velocity;
x,	axial distance along tube;
β_u,	eigenvalues of equation (2);
κ,	thermal conductivity of fluid;
μ,	fluid viscosity;
ρ,	fluid density.

INTRODUCTION

SEVERAL theoretical analyses have been made with regard to fluid flow in a circular tube with the fluid containing internal heat sources [1–4]; however, no experimental studies have apparently been undertaken that would provide assurance of the physical significance of these theoretical solutions. For this reason, the experimental study about to be described was conducted. This investigation dealt with laminar tube flow with uniform internal heat generation.

DESCRIPTION OF APPARATUS

A schematic diagram of the experimental apparatus is shown in Fig. 1. Water containing an electrolyte (NaCl, the salt being added to increase the electrical conductivity of the fluid) was pumped from the reservoir through the entrance section and into the test section, where the fluid was heated. After passing through the test section, the fluid flowed through mixing baffles and then through a heat exchanger and finally returned to the reservoir for recirculation. Between the entrance and test sections and between the test and mixing sections there was a ring electrode whose inner radius matched that of the sections. Electrode and glass sections were separated from one another by neoprene gaskets 0·125 in thick.

The entrance section consisted of 1·025-in i.d

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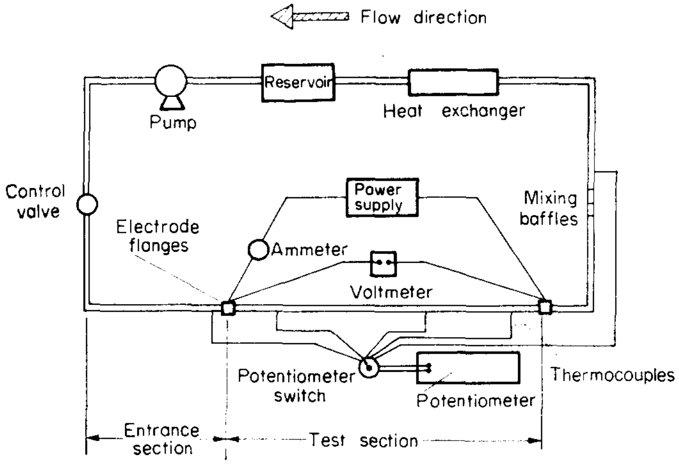

FIG. 1. Schematic diagram of experimental apparatus.

glass tubing, 0·125-in wall thickness and approximately 53 diameters in length, and was provided with a thermocouple well and a small opening in the wall for the insertion of a hypodermic needle into the fluid stream. This section was covered with 1-in fiberglass blanket insulation.

The test section consisted of 1·025-in i.d. glass tubing with 0·125-in wall thickness and was approximately 96 diameters in length. Thermocouple wells were located 7, 13, 25, 49, 72 and 90 in, respectively, from the entrance to the section. This section was likewise covered with 1-in fiberglass blanket insulation.

The mixing section consisted of 1·025-in glass tubing (wall thickness, 0·125 in) and was approximately 4 diameters in length. This section contained three glass disks, cemented into the tube and aligned perpendicular to the flow. These disks contained several holes of various diameters, and the holes were misaligned from disk to disk to induce mixing of the fluid. A thermocouple well was situated downstream of the mixing baffles. This section was insulated with 1-in fiberglass blanket.

A counter-flow heat exchanger was used to cool the circulating fluid after it was heated in the test section. This exchanger consisted of two concentric copper tubes, 3 ft in length and with an inside diameter of 0·5 and 1·0 in, respectively. Because of the presence of a mild salt-water solution and a small internal electrical current, the inner surface of the smaller tube was coated with a corrosion-resistant paint.

The reservoir consisted of a 50-gal-capacity drum, whose interior was likewise coated with corrosion-resistant paint. A standard laboratory $\frac{1}{15}$-h.p. pump was used to circulate the fluid. A manually operated control valve, placed upstream of the entrance section, was used to regulate the flow. The three glass sections were secured to one another by bolts through special flanges.

Electrical power, used for heating the fluid, was obtained from a 400-c/s power supply. This supply had a maximum output of 208 V (a.c.), 8·3 A at 400 c/s, single-phase. A 2:1 transformer was used to boost the voltage being supplied to the test section, the voltage being controlled by a 230-V Variac situated on the input side of the transformer. Power was applied to the fluid via two graphite ring electrodes. These electrodes were ring-shaped, with 1·00 in i.d., 1·75 in o.d., and were $\frac{3}{8}$ in wide.

Preliminary experiments with 60 c/s a.c. power indicated electrolysis of the water even at moderate voltages, and dictated the need for higher frequency. The choice of 400-c/s a.c. power was a matter of availability and convenience. For the voltage used in this experiment (265 V), no electrolysis was encountered. The electrodes were machined from graphite, because several different metals, when used as electrodes,

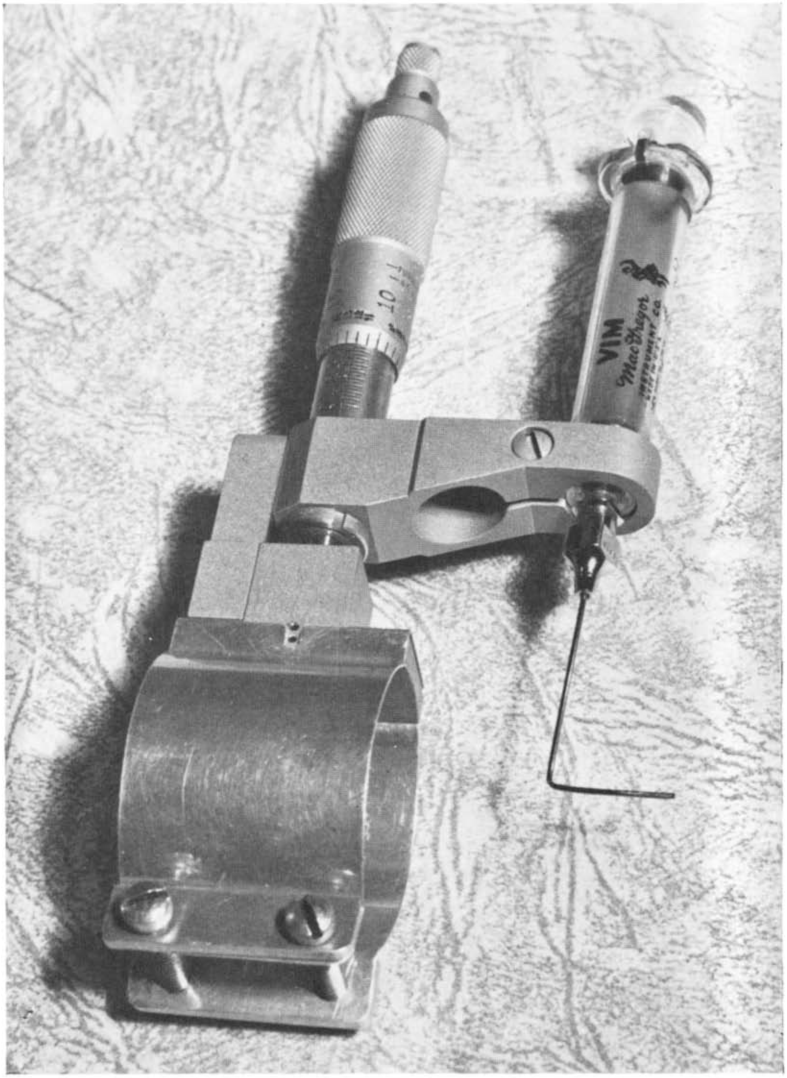

FIG. 3. Velocity probe.

corroded rather badly at high voltages. The electrodes were machined into rings so that the fluid would flow through them without disturbance, and were made thin in order to minimize end effects. Each electrode was wrapped with a strip of copper, to which were attached power leads. The voltage supplied to the test section was measured by a Simpson voltmeter while the current was measured with a 25–500 c/s laboratory ammeter.*

Inasmuch as an electric current was passing *through* the fluid, the return loop of the system constituted another branch for current flow, and hence the electrical resistance of that branch had to be made large relative to the test section in order to minimize current "leakage". This was accomplished by (1) lengthening the return loop relative to the test section (through the addition of long lengths of rubber tubing) and by (2) reducing the diameter of the return-loop tubing. It was estimated that the resistance of the return loop was about twenty-five times that of the test section. Likewise, all metallic instruments and equipment were isolated from ground to prevent current leakage.

Eight thermocouples were located as mentioned earlier throughout the equipment. They consisted of 30-gage iron–constantan wire and were placed in wells $\frac{1}{16}$ inch in diameter. The wells extended completely through the glass wall so that the thermocouples were in intimate contact with the surface of the fluid. Details of the installation can be seen in Fig. 2. Thermocouple tips were spot-welded for protection against corrosion. Because of the large voltage imposed upon the system, as well as the current flow past the thermocouples, it was necessary to "isolate" each thermocouple from the others when a reading was being taken. This "isolation" was accomplished through the use of a two-gang non-shorting tap switch. A Leeds and Northrup 8693 temperature potentiometer, reading directly in degrees Fahrenheit, was used for measurement of wall and bulk temperatures, with a precision of about ±0·1 degF. All thermocouples were calibrated in place.

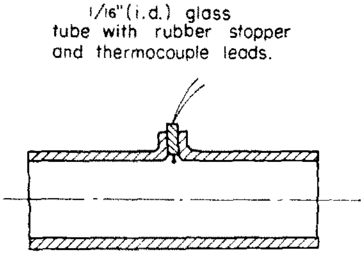

FIG. 2. Detail of fluid thermocouple installation (not to scale).

Fluid flow rates were determined by measuring the weight of liquid accumulated in a certain time.

Velocity distribution was determined by a dye-displacement technique. A traversing mechanism was located in the entrance section (a hydrodynamic starting length of 50 diameters was allowed for the fluid). The mechanism consisted of a 20-gage stainless-steel hypodermic needle inserted perpendicular to the flow, with a portion of the needle bent to face downstream. A hypodermic syringe was attached to the needle. The syringe was filled with a dye which was injected in droplets into the fluid stream through the needle. The time required for a droplet to move a prescribed distance along the flow direction was noted. The radial position of the needle tip was accurately measured by the use of a micrometer head, with divisions of 0·001 in, rigidly secured to the syringe and needle. Details of this traversing mechanism can be seen in Fig. 3.

Knudsen and Katz [5] present curves comparing experimental and theoretical velocity profiles for laminar Newtonian flow in circular tubes. Excellent agreement of experimental results with the well-known parabolic velocity-distribution equation is demonstrated. Since this same profile was expected for the present investigation, velocity distribution was not measured with as much precision as temperature distribution.

PROCEDURE

Water containing a small amount of sodium chloride (the salt being added to increase the electrical conductivity of the fluid) was circulated through the system, initially at a rather large

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* These meters were calibrated by the Electrical Standards Laboratory, Department of Engineering, University of California, Los Angeles.

flow rate in order to remove air bubbles entrained in the fluid; when this removal was accomplished, the desired flow rate was set through the use of the control valve and fluid weight measurements. After flow-rate stabilization had been obtained, and before power to the test section was turned on, a radial velocity traverse was made. Pai [6] has shown that for the type of fluid and power input used in this investigation, the magnetohydrodynamic effect on the velocity distribution can safely be neglected. Following the completion of the velocity traverse, the needle was retracted to avoid disturbance of the fluid stream, and power to the test section was turned on. Simultaneously, city water drawn from the central supply mains was supplied to the heat exchanger, to serve as a coolant. This coolant was rejected to a drain. The voltage and the coolant flow were adjusted until all temperatures throughout the system were invariant with time, and the bulk temperature of the fluid flowing out of the mixing section was 10–11 degF above the bulk temperature of the fluid entering the test section. With such small differences, physical properties of the fluid were essentially constant. As soon as this condition was reached, ammeter, voltmeter and thermocouple readings were obtained. These readings were repeated, to assure invariance with time of the data.

ANALYSIS OF DATA

The experimental data collected are shown in Figs. 4–6 in comparison with the theoretical results.

The velocity-distribution data is compared with the (expected) parabolic velocity-distribution equation

$$f(r) \equiv u(r)/\bar{u} = [1 - (r/r_0)^2] \qquad (1)$$

in Fig. 4. It is seen that the data can be represented by equation (1) within the limits of experimental error.

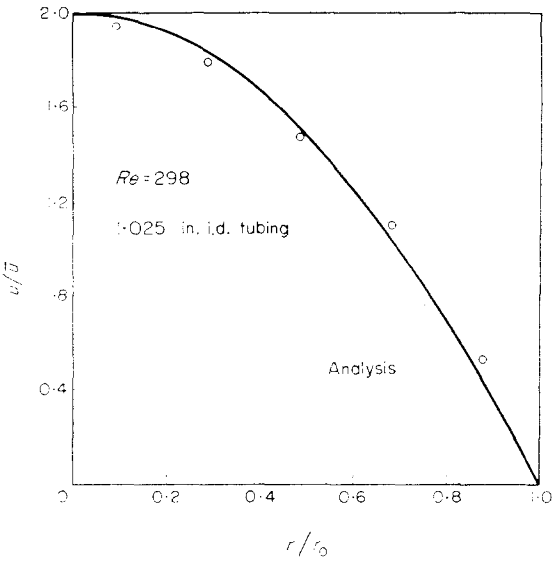

FIG. 4. Velocity distribution for laminar flow in a round tube.

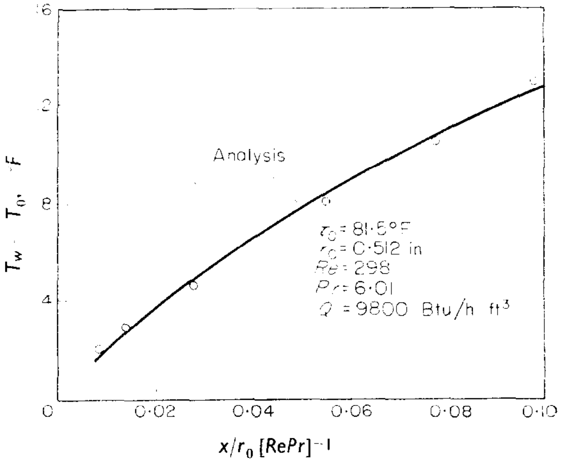

FIG. 5. Wall temperature variation for uniform internal heat source in an insulated tube (laminar flow).

Because of the high electrical resistivity of the fluid as well as a sensibly constant value for the resistivity, it can be concluded that the heat source is uniform longitudinally (except in the immediate neighborhood of the ring electrodes; however, these non-uniform regions constitute only about 1 per cent of the over-all tube length, and can be neglected). There remains to consider the possibility of "skin effect", which would give rise to a radical variation of the heat-source distribution in the tube. Calculations based on equations given by Ramo and Whinnery [7] for the current distribution in an electrical conductor of circular cross-section show that, for the electrical properties of the conductor and the

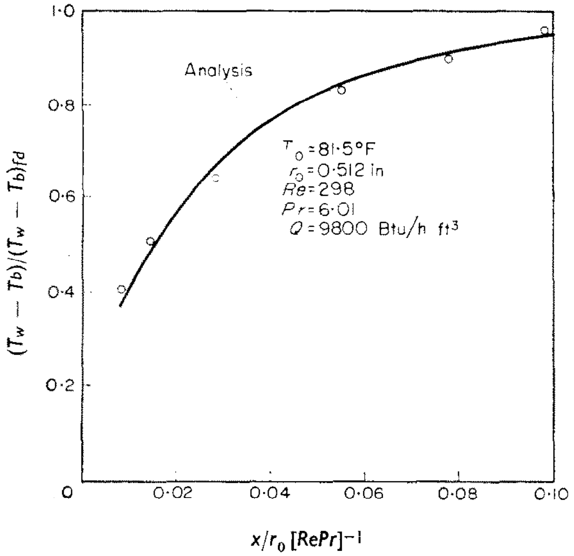

FIG. 6. Wall-to-bulk temperature variation for uniform internal heat source in an insulated tube (laminar flow).

frequency of current distribution chosen for this experiment, the skin effect is negligible, i.e. the current has a uniform distribution of the cross-section. Hence it can be concluded that the heat source is uniform throughout the test section.

Because of the presence of a uniform heat source, the wall-temperature distribution is determined from the theoretical relation given by Sparrow and Siegel [2]:

$$(T_w - T_0)/(Qr_0^2/\kappa) = 2(x/r_0)(1/RePr) + (1/16) + \sum_{u=1}^{\infty} C_u R_u(1) \exp\,[-\,\beta_u^2(x/r_0)(1/RePr)]. \quad (2)$$

Values of C_u, $R_u(1)$ and β_u have been tabulated by Sparrow and Siegel [2]. In Fig. 5, data is shown for the wall-temperature distribution and is compared with a curve calculated from equation (2) (fluid properties were obtained from [8] and [9] and were evaluated at the length-mean bulk temperature). Comparison between theory and experiment is excellent.

An additional check on the theory is provided by consideration of the expression for the fluid-bulk temperature rise due to a uniform internal heat source, given by Sparrow and Siegel [2]:

$$T_b(x) - T_0 = (Qr_0^2/\kappa)[2(x/r_0)(1/RePr)]. \quad (3)$$

Using the data presented in Fig. 5 and setting $x = L$ ($= 96$ in),

$$(Qr_0^2/\kappa) = 50{\cdot}9\,^\circ\mathrm{F}; \quad (L/r_0)(1/RePr) = 0{\cdot}105$$

and hence $T_b(L) - T_0 = 10{\cdot}7$ degF. The measured rise over the length of the test section was 10·8 degF.

The wall-to-bulk temperature variation for a uniform heat source is determined from the data [the bulk temperature was evaluated from equation (3)] and compared in Fig. 6 with the theoretical variation presented by Sparrow and Siegel [2]:

$$(T_w - T_b)/(T_w - T_b)_{fd} = 1 + 16 \sum_{u=1}^{\infty} C_u R_u(1) \exp\,[-\,\beta_u^2(x/r_0)(1/RePr)] \quad (4)$$

where $(T_w - T_b)_{fd}$ is the temperature difference in the fully developed situation ($x \to \infty$) and is given by

$$(T_w - T_b)_{fd} = (1/16)(Qr_0^2/\kappa). \quad (5)$$

CONCLUSION

Results of the experimental study of temperature distribution for laminar Newtonian fluid flow in an insulated tube with uniform internal heat generation are in excellent agreement with theory.

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Résumé—Une recherche expérimentale sur l'écoulement laminaire dans un tube circulaire avec source de chaleur intérieure a été entreprise. Les mesures de température de paroi faites dans le cas d'une source de chaleur uniforme à l'intérieur d'un tube isolé, sont en très bon accord avec l'analyse théorique faite par Siegel et ses collaborateurs.

Zusammenfassung—Die laminare Flüssigkeitsströmung mit inneren Wärmequellen wurde in Rohren von Kreisquerschnitt experimentell untersucht. Die am isolierten Rohr durchgeführten Messungen der Wandtemperatur bei gleichmässiger innerer Wärmeerzeugung zeigten gute Übereinstimmung mit einer theoretischen Analyse von Siegel und anderen.

Аннотация—В статье описываются эксперименты по исследованию ламинарного течения в круглой турбе с внутренними источниками тепла. Проведены измерения температуры стенки для однородного внутреннего источника тепла в изолированной трубе и показано, что экспериментальные результаты хорошо согласуются с теоретическими данными Сигеля и др.